Sohag University
Faculty of Engineering

Strength of Materials
Civil 1st Year

Sheet 4 : Bending Stresses

1- A bending test was carried out on a steel beam having a cross section 4x8 cm and 80 cm span length. The beam is **centrally loaded.** the load (P), and the middle deflection (y) were as follows :

P(kg)	200	400	600	800	920	880	910	1000	1350	1600	1750	1800	1810	1750	1600
δ(mm)	0.5	1	1.5	2	2.15	2.3	2.4	2.5	3	3.5	4	4.5	5	5.5	6

Draw the load deflection diagram and determine:

- **Proportional limit stress.**
- **Modulus of elasticity.**
- **Modulus of rapture.**
- **Elastic energy stored in beam.**
- **Modulus of resilience.**

1- (Proportional limit stress) = σ_{pr}

I_x = (3العمودى * الموازى) /12 = $(4*8^3) / 12 = 170.67\ cm^4$

$Y_{max} = 4$ cm , $\because$ **(Proportional limit force)** $P_{pro} = 800$ kg

$\therefore M_{pro(max)} = P_{pro} * L(span) / 4 = (800*80) / 4 = 16000$ kg.cm

$\therefore \sigma_{pr} = (M_{pro} * Y_{max}) / I_x = (16000 * 4) / 170.67 = 374.993\ kg/cm^2$

2- **(Modulus of elasticity)** E (λ)

at P = 200 kg (deflection) $\delta = 0.5$ mm

$\because \delta = (P * L^3) / (48 * E * I_x)$

$\therefore$ $0.5/10 = (200*80^3)/(48*E*170.67)$

$\therefore$ $E = 249995.12\ kg/cm^2$

3- (Modulus of rapture) = **σ_{ult} =(ultimate stress)**

$P_{pro} = 1820$ kg $\therefore$ $M_{max} = P_{ult} * L/4 = 18\cancel{2}0*80/4 = 36\cancel{4}00$ kg.cm

$\therefore$ $\sigma_{ult} = M_{max} * Y_{max} / I_x = (36400*4) / 170.67 = \cancel{853.11}$ kg/cm^2

848.4

4- (Elastic energy stored in beam) = R

at $P_{pro} = 800$ kg , δ_{pro}

$R = 0.5 * P_{pro} * \delta_{pro} = 0.5*800*0.2 = 80$ kg.cm

5- (Modulus of resilience) = M.R

$M.R = 0.5 * \sigma_{pr} * \varepsilon_{pr} = R / volume = 80 / (8*4*80) = 0.03125$ kg/cm^2

2- A simply supported beam with a rectangular cross section XxY, Loaded by a distributed load W along its span L.
How is the stress changed by reducing to half each of:

- Distributed load W.
- Length of span.
- width of the beam.
- Depth of the beam.

$Y_{max} = y/2$, $M_{(max)} = w*L^2/8$, $I_x = \frac{x*y^3}{12}$, $A = x*y$

$\sigma_1 = M * Y_{max} / I_x = (w*L^2/8) * 0.5\ y*(12/xy^3) = (3/4)*(wL^2/xy^3) = (3/4)\lambda$

Where : $\lambda = (wL^2/xy^3) = (4/3)\,\sigma_1$ constant

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1- When : $w_2=0.5\ w_1$

$\therefore \sigma_2 =$

$(3/4)*(0.5W*L2)/(xy2)=(3/8)*\ (wL^2/xy^2)=(3/8)\lambda=(3/8)*(4/3)\ \sigma_1=0.5\ \sigma_1$

2-When : $L_2=0.5\ L_1$

$\therefore \sigma_2=(3/4)*(w*(0.5L)^2)/(xy^2)\ =(3/4)*(w*0.25L^2)/(xy^2)\ =$

$(3/16)*(wL^2/xy^2)=(3/16)\ \lambda=(3/16)*(4/3)\ \sigma_1\ =0.25\ \sigma_1$

3-When : $X_2=0.5\ X_1$

$\therefore \sigma_2=(3/4)*(w*L^2)/(0.5xy^2)=(3/2)*(wL^2/xy^2)=(3/2)\ \lambda=(3/2)*(4/3)\ \sigma_1\ =$

$2\ \sigma_1$

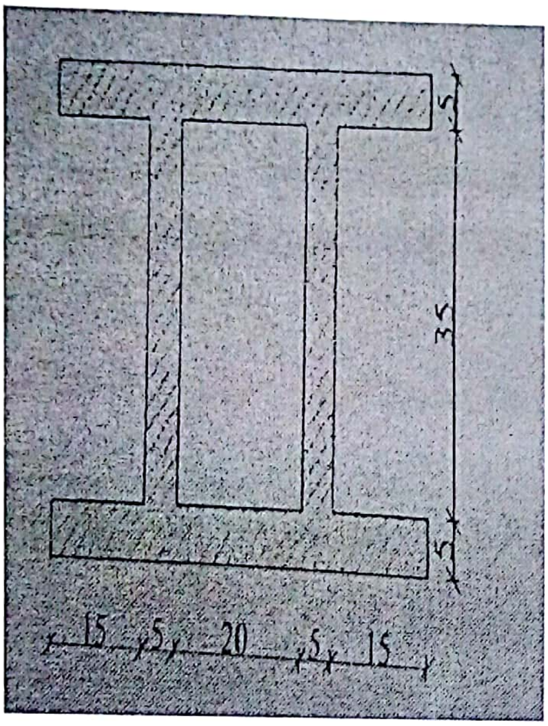
4-When : $Y_2=0.5\ Y_1$

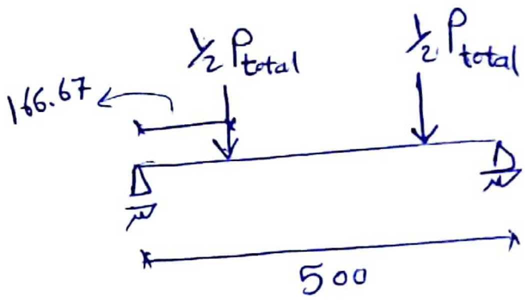
$\therefore \sigma_2=(3/4)*(w*L^2)/(x*(0.5y)^2)=(3/4)*(w*L^2)/(0.25xy^2)=3*(wL^2/xy^2)$

$=3\ \lambda=3*(4/3)\ \sigma_1\ =4\ \sigma_1$

3-Bending test was carried out on a beam having **H**-cross section as shown in figure, the beam is simply supported at point's 500 cm apart, with two-third points loading.Strain guage was attached to the beam at the maximum deflection location. If the modulus of elasticity is 8x10^5 kg/cm^2.

- What load is required to give max. elastic strain reading of 0.003 ?
- The maximum elastic deflection.
- Elastic strain energy stored in beam.





$E = 8*10^5\ kg/cm^2$, $\epsilon = 0.003$, $Y_{max} = 22.5\ cm$

${}_{max}B.M = 1/2*P_{total}*166.67 = 83.335P_{total}\ t.cm$

$I_x = [\,2\{((60*5^3/12)+60*5*20^2)+(5*35^3/12)\}] = 276979.167\ cm^4$

1- $\because \sigma = E*\epsilon = (8*(10^5))*0.003 = 2400\ kg/cm^2$

$\because \sigma = (M_{max}*Y_{max})/I_x = (8.3335*P_{total}*1000*22.5)/276979.167 = 2400$

$\therefore 6.77\ P = 2400 \therefore P_{total} = 354.51\ ton \therefore P/2 = 177.255\ ton$

2- (deflection)δ

$\delta_{max} = (23/648)*(PL^3/EI)$

=(23/648)*(177.255*1000*(500^3))/(8*(10^5)*276979.167)

=3.55 cm

3- (elastic strain energy stored in the beam) R

$R = 0.5 * P * \delta_{max}$ =0.5*177.255*3.55=314.63 t.cm

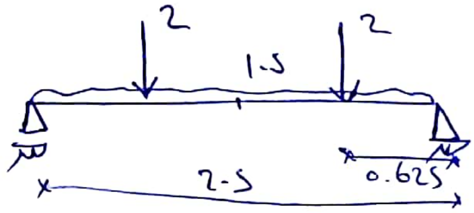
= 314627.625 kg.cm

4- Determine the cross section dimensions of a 2.50 m simply supported beam with two point loads applied at the **fourth points** under a total load of 4 tons and a distributed load equal 1.5 t/m.The modulus of rupture is 800kg/cm^2 ,with a depth equal to four times the width. Use design stress based on the modulus of rupture with a factor of safety equals 3.

$_{max}$B.M = 1.25+1.172= 2.422 t.m , Y_{max}=2b cm

$I_x = (b*(4b)^3)/12 = 5.33b^4\ cm^4$,modulus of rapture = σ_{ult}=800kg/cm^2

f.o.s = $\sigma_{ult} / \sigma_{all}$ = 800 / σ_{all} = 3



$\therefore \sigma_{all}$ = 266.67 kg/cm^2

$\because \sigma_{all} = (M_{max} * Y_{max}) / I_x = (2.422*(10^5)*2b)/(5.33b^4) = 266.67$

$(90881.80113/b^3) = 266.67$

$b^3 = 340.8025$

$\therefore$ b = 6.985019 ≈ 7 cm $\therefore$ d = 4*7 = 28 cm

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$\therefore\ b = 6.985019 \approx 7 \text{ cm} \quad \therefore\ d = 4.7 = 28 \text{ cm}$